KAUNO TECHNOLOGIJOS UNIVERSITETAS
INFORMATIKOS FAKULTETAS

Modulio P170B115 „Skaitiniai metodai ir algoritmai“

Laboratorinio darbo ataskaita
Pirmas laboratorinis darbas

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Turinys

1. Užduoties variantas

Variantas: 20
Metodai: Stygų, Niutono (liestinių)

$$f(x) = 1.34x^4 - 16.35x^3 + 65.13x^2 - 83.45x - 3.27$$
$$g(x) = \sin(x) - \frac{\ln(x)}{2} + 0.1; 0.1 \le x \le 10$$
$$h(x) = -8\sin(x) + 32\sin(10x) - 10;\ 0 \le x\ \le 1$$

2.1 Dalis

Išspręskite netiesines lygtis, kai lygties funkcija yra daugianaris $f(x) = 0$ ir transcendentinė funkcija $g(x) = 0$.

2.1. Daugenario šaknies intervalo nustatymas

„Grubaus“ metodo funkcija:

```python
def grubus_R(LF_terms):
    return 1 + max(abs(ai) for ai in LF_terms[:-1]) / LF_terms[-1]
```

„Tikslesnio“ metodo funkcija:

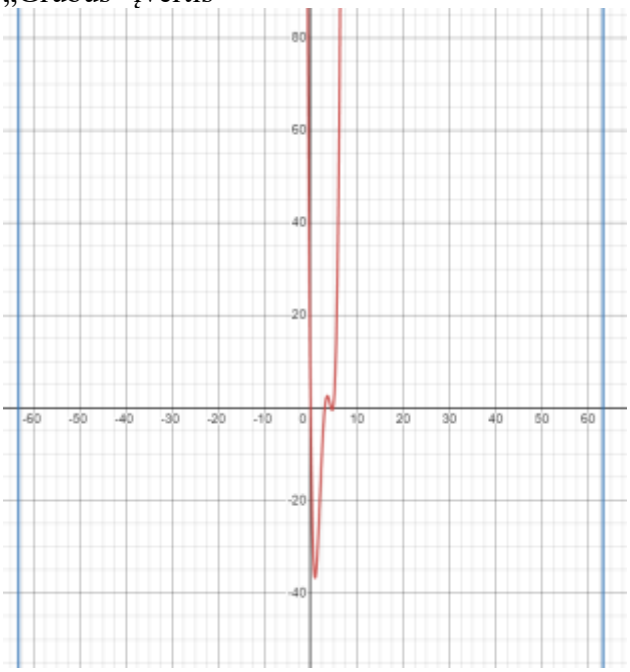
```python
def mul_every_by(numbers: list[float], scalar: float):
    return list(map(lambda a: scalar*a, numbers))

def tikslesnis_R_range(LF_terms: list[float]):
    LF_terms = LF_terms.copy()
    if LF_terms[-1] < 0:
        # f(x) => -f(x)
        LF_terms = mul_every_by(LF_terms, -1)


    k = len(LF_terms) - max(i for i, a in enumerate(LF_terms[:-1]) if a < 0) - 1
    B = max(abs(a) for a in LF_terms[:-1] if a < 0)
    R_teig = 1 + (B / LF_terms[-1]) ** (1/k)

    # f(x) => f(-x)
    for i in range(1, len(LF_terms), 2):
        LF_terms[i] *= -1

    if len(LF_terms) % 2 == 0:
        # f(x) => -f(x)
        LF_terms = mul_every_by(LF_terms, -1)

    if any(True for a in LF_terms[:-1] if a < 0) > 0:
        B = max(abs(a) for a in LF_terms[:-1] if a < 0)
        k = len(LF_terms) - max(i for i, a in enumerate(LF_terms[:-1]) if a < 0) - 1
        R_neig = 1 + (B / LF_terms[-1]) ** (1/k)
    else:
        R_neig = 0

    return (R_neig, R_teig)
```

Bendras intervalo įvertis naudojant „grubų“ ir „tikslesnį“:

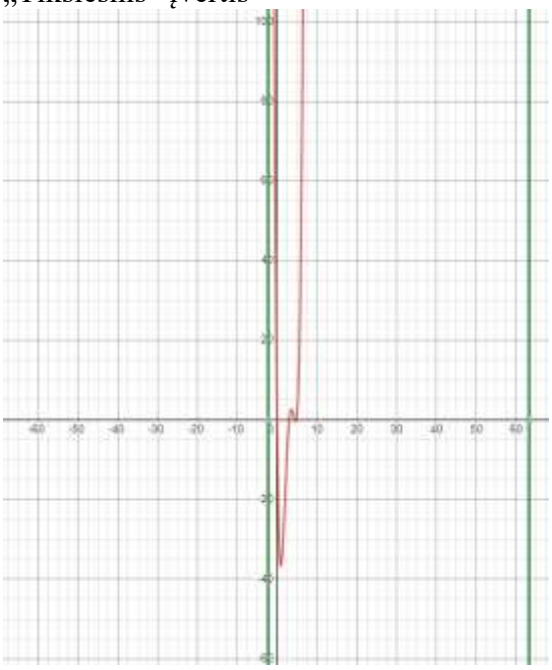
```python
def roots_range(LF_terms: list[float]):
    R = grubus_R(LF_terms)
    R_neig, R_teig = tikslesnis_R_range(LF_terms)
    return (-min(R, R_neig), min(R, R_teig))
```

Programos rezultatas f(x) bendro intervalo įverčiui:
Nuo -2.249 iki 63.276.

„Grubus“ įvertis



„Tikslesnis“ įvertis



2.2. Šaknų atskyrimo intervalų radimas naudojant skenavimo algoritmą

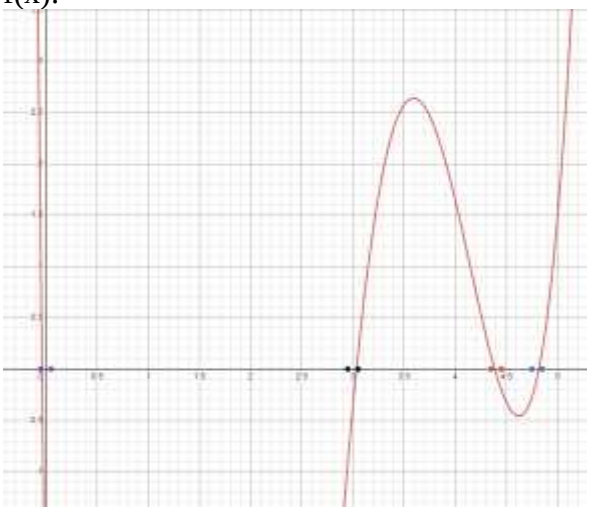
Skenavimo funkcija:

```
def scan_root_ranges(LF, x1, x2, step=0.1):
    x = x1
    while x < x2:
        if np.sign(LF(x)) != np.sign(LF(x+step)):
            yield (x, x+step)
        x += step
```

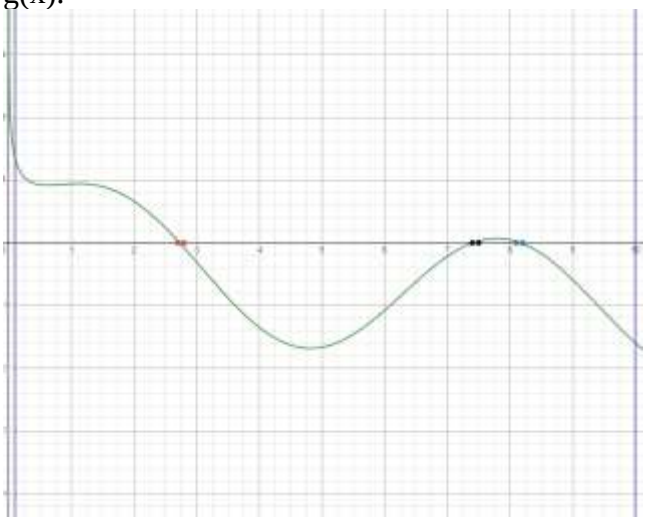
Programos rezultatas f(x) ir g(x) šaknų atskyrimo intervalai naudojant skenavimo algoritmą, skenavimo žingsnio dydis yra 0.1:

Funkcija	Intervalo pradžia	Intervalo pabaiga
f(x)	-0.04985	0.05014
	2.95014	3.05014
	4.35014	4.45014
	4.75014	4.85014
g(x)	2.7	2.8
	7.4	7.5
	8.1	8.2

f(x):



g(x):



2.3. Šaknies tikslinimas

Naudojant stygų metodą tikslinimiui funkcija su visualizacija:

```
def solve_stygos(F, min_x, max_x, view_x, max_iterations=30, epsilon=1e-6, animation_delay=0.05):

    numpy_F = lambda x: F(NumpyMathLibrary, x)

    textbox = create_scroll_textbox(140, 20)
    textbox_append_line(textbox, "-------- Stygu metodas -------------")

    my_plot = MyPlot(animation_delay)
    my_plot.plot_function(view_x[0], view_x[1], numpy_F)
    my_plot.add_zero_line(view_x[0], view_x[1])

    textbox_append_line(textbox, "Saknies reiksmes tikslinimas Stygu metodu:")

    results = []

    for x1, x2 in scan_root_ranges(numpy_F, min_x, max_x):
        start_x1 = x1
        start_x2 = x2
        my_plot.subplot.plot([x1, x2], [0, 0], 'mo--')
        plt.draw()

        f1 = numpy_F(x1)
        f2 = numpy_F(x2)
        my_plot.update_edge_points(x1, f1, x2, f2)

        for i in range(1, max_iterations + 1):
            if f2 == 0:
                f2 += epsilon/2
            k = np.abs(f1 / f2)
            x_middle = (x1 + k * x2) / (1 + k)  # stygos nulio taskas
            middle_value = numpy_F(x_middle)

            my_plot.clear_connecting_line()
            my_plot.update_middle_point(x_middle, middle_value)
```

```
            if np.sign(f1) != np.sign(middle_value):
                x2 = x_middle
                f2 = middle_value
            else:
                x1 = x_middle
                f1 = middle_value

            my_plot.clear_middle_point()
            my_plot.update_edge_points(x1, f1, x2, f2)

            tksl = get_guess_accuracy(x1, x2, f1, f2, epsilon)
            textbox_append_line(textbox, f"{i:2d}. x1={x1:-20g}, x2={x2:-20g}, f1={f1:-20g}, f2={f2:-20g}, tikslumas={tksl:-
20g}")

            if tksl < epsilon:
                break

        tksl = get_guess_accuracy(x1, x2, f1, f2, epsilon)
        if tksl < epsilon:
            print(f"Isspresta: x={x1:g}, f={f1:g}, tikslumas={tksl:g}")
        else:
            print("Tikslumas nepasiektas")

        results.append([start_x1, start_x2, x1, tksl, i])

    return results
```

Naudojant Niutono liestinių metodą tikslinimiui funkcija su visualizacija:

```
def solve_liestiniu(F, min_x, max_x, view_x, epsilon=1e-8, animation_delay=0.5):
    sympy_F = lambda x: F(SympyMathLibrary, x)
    numpy_F = lambda x: F(NumpyMathLibrary, x)

    x_symbol = sympy.symbols("x")
    DF_symbolical = sympy_F(x_symbol).diff(x_symbol)
    DF = sympy.lambdify((x_symbol), DF_symbolical)

    x=np.arange(view_x[0], view_x[1], 0.05); y=numpy_F(x)

    plt.xlabel("x"); plt.ylabel("y"); plt.plot(x, y, 'b'); plt.grid()

    results = []

    for x1, x2 in scan_root_ranges(numpy_F, min_x, max_x):
        start_x = (x1+x2)/2

        xi = start_x
        iterations = 0
        while np.abs(numpy_F(xi)) > epsilon:
            xi_bef = xi
            xi = xi - (1 / DF(xi)) * numpy_F(xi)
            iterations += 1

            plt.plot([xi_bef], numpy_F(xi_bef), 'ob')
            plt.plot([xi], [0], 'or')
            plt.plot([xi_bef, xi], [numpy_F(xi_bef), 0], 'r-')
            plt.plot([xi, xi], [0, numpy_F(xi)], 'g--')
            plt.draw()
            plt.pause(animation_delay)

        results.append((x1, x2, xi, np.abs(numpy_F(xi)), iterations))

    return results
```

Programos šaknų radimo rezultatas naudojant Niutono (liestinių) ir stygų metodus:

Funkcija	Metodas	Pradinis intervalas	Kirtimo taškas	Tikslumas	Iteracijos
f(x)	Stygų	[-0.04985; 0.05014]	-0.0380447	7.51696e-07	10
		[2.95014; 3.05014]	3.03508	2.19478e-16	15

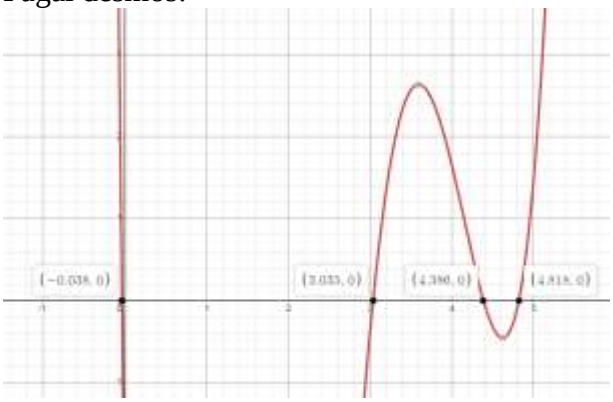
		[4.35014; 4.45014]	4.38623	1.22305e-13	9
		[4.75014; 4.85014]	4.81822	5.71446e-15	14
	Niutono (liestinių)	[-0.04985; 0.05014]	-0.0380447	6.68674e-11	3
		[2.95014; 3.05014]	3.03508	1.41154e-11	3
		[4.35014; 4.45014]	4.38623	1.52323e-13	3
		[4.75014; 4.85014]	4.81822	2.16783e-10	3
g(x)	Stygų	[2.7; 2.8]	2.72811	1.62783e-16	8
		[7.4; 7.5]	7.4	0.000372351	30
		[8.1; 8.2]	8.17066	8.13851e-10	10
	Niutono (liestinių)	[2.7; 2.8]	2.72811	8.13851e-10	2
		[7.4; 7.5]	7.40551	3.70581e-11	3
		[8.1; 8.2]	8.17066	8.10185e-14	3

Iš programos rezultatų galima daryti kelias išvadas:

- Stygų metodas nepasieks kirtimo taško jeigu vienas iš pradinio intervalo galų yra labai arti kirtimo taško
- Stygų metodas geriau veikia kuo arčiau vidurio kirtimo taškas yra pradinio intervalo vidurio, suras kirtimo tašką per mažiau iteracijų
- Niutono (liestinių) metodas suras kirtimo tašką per mažiau iteracijų, nes tame metode funkcijos liestinė gali tiksliai pasakyti, kaip reikia x keisti, kad artėti prie y=0

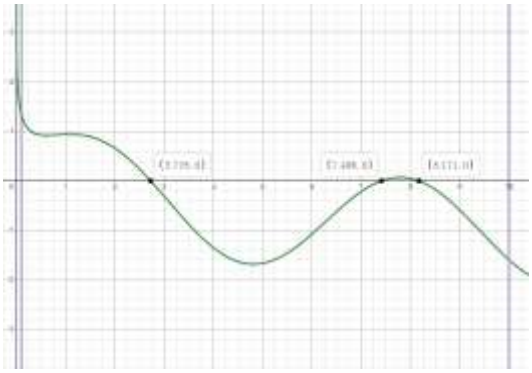
2.4. Gautų šaknų patikrinimas

f(x) šaknų tikrinimas:
Pagal desmos:

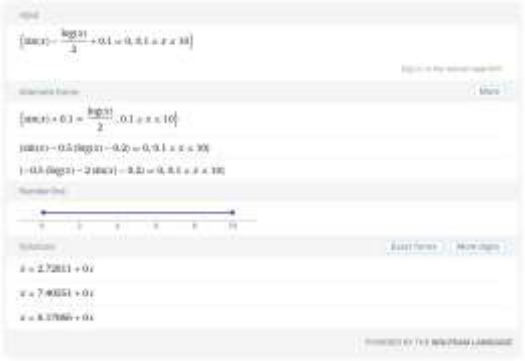


Pagal WolframAlpha:

g(x) šaknų tikrinimas:
Pagal desmos:



Pagal WolframAlpha:

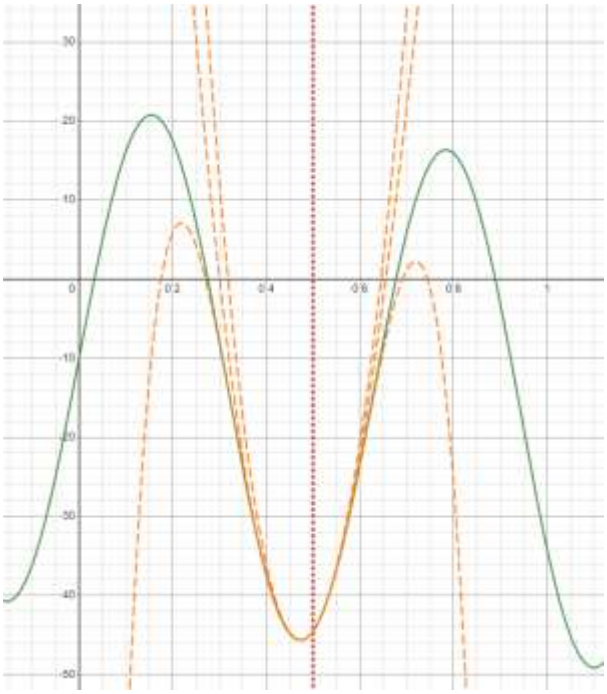


3.2 Dalis

Pateiktą funkciją $h(x)$ išskleiskite Teiloro eilute (TE) nurodyto intervalo vidurio taško aplinkoje. Nustatykite TE narių skaičių, su kuriuo visos TE šaknys esančios nurodytame intervale, skiriasi nuo funkcijos $h(x)$ šaknų ne daugiau negu |1e-4|. Tiek pateiktos funkcijos $h(x)$ šaknis, tiek TE šaknis raskite antru iš pirmoje dalyje realizuotų skaitinių metodų (Niutono).

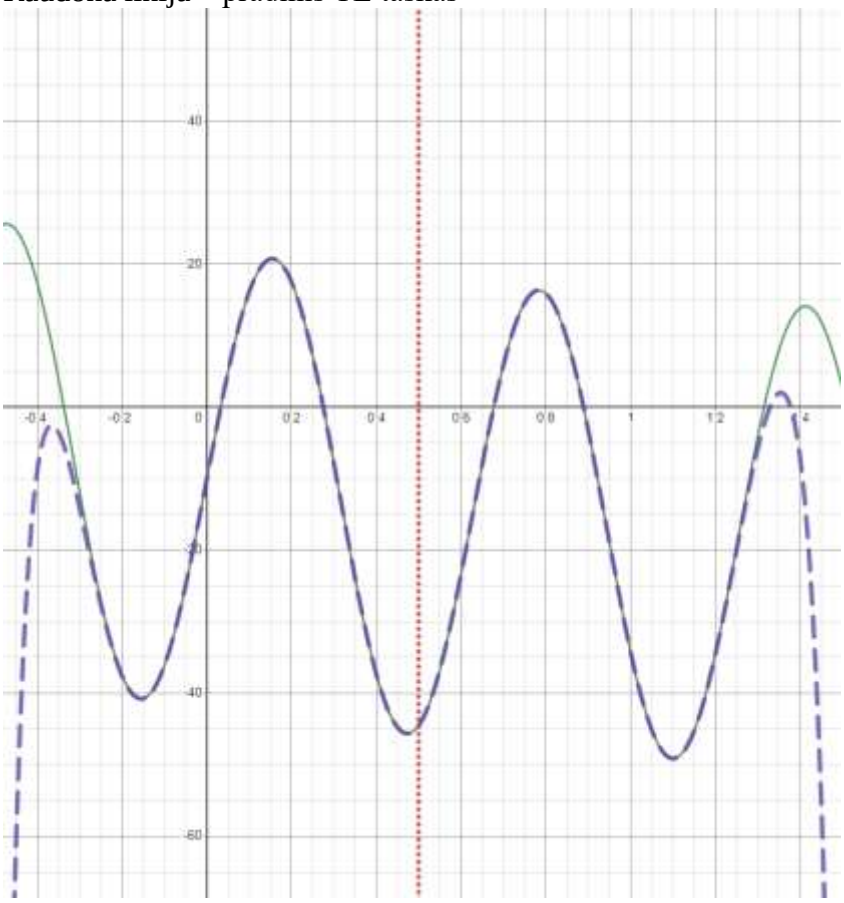
3.1. TE tarpinės funkcijos

Žalia linija – h(x)
Raudona linija – pradinis TE taškas
Geltonos linijos – 3, 4 ir 5 eilės teiloro eilutės



3.2. TE tikslumo atvaizdavimas grafiškai

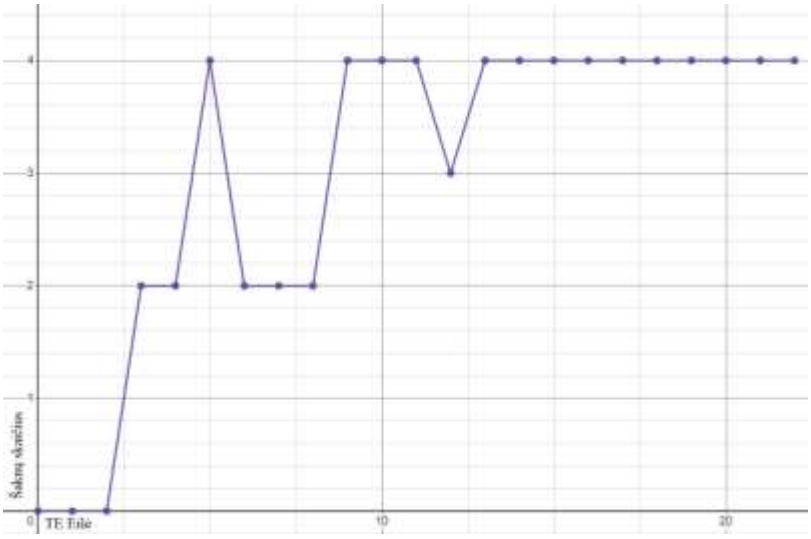
Žalia linija – h(x)
Violetinė linija – 22 TE funkcija
Raudona linija – pradinis TE taškas



3.3. Analitinė išraiška

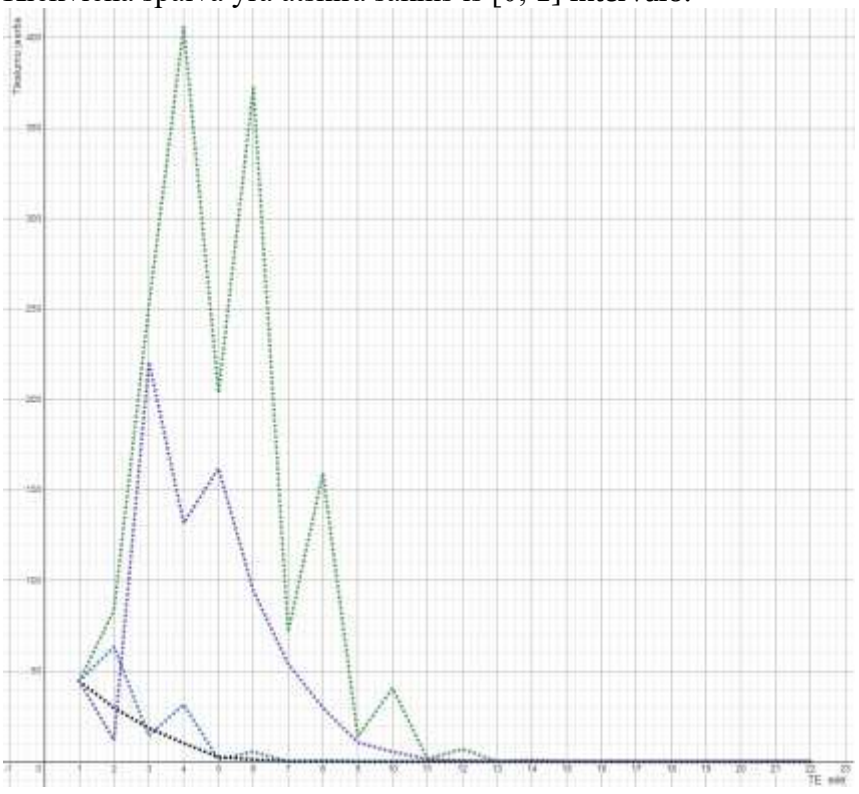
$$
\begin{aligned}
&(177.667303615504\,x^{21}) + (-3126.78135854510\,x^{20}) \\
&+ (21194.0774704520\,x^{19}) + (-77565.9669162660\,x^{18}) \\
&+ (173706.571367708\,x^{17}) + (-257557.626211679\,x^{16}) \\
&+ (289245.368829960\,x^{15}) + (-307120.810591593\,x^{14}) \\
&+ (297668.883815680\,x^{13}) + (-163671.552571374\,x^{12}) \\
&+ (10591.1193766985\,x^{11}) + (-42118.7003803260\,x^{10}) \\
&+ (104538.523032389\,x^{9}) + (-5296.17992616654\,x^{8}) \\
&+ (-62071.4960597741\,x^{7}) + (-312.183401104267\,x^{6}) \\
&+ (26721.8934508589\,x^{5}) + (-7.70217548539222\,x^{4}) \\
&+ (-5331.18733792747\,x^{3}) + (-0.0610647714044319\,x^{2}) \\
&+ (312.002912257649\,x^{1}) + (-10.0000662682033\,x^{0})
\end{aligned}
$$

3.4. Šaknų skaičiaus priklausomybė nuo TE eilės



3.5. Šaknų tikslumo priklausomybė nuo TE eilės

Kiekviena spalva yra atskira šaknis iš [0; 1] intervalo.



4. Programinis kodas

```
import math
import matplotlib.pyplot as plt
import numpy as np
import tkinter as tk
import sympy
from tabulate import tabulate

class MyPlot:
    def __init__(self, animation_delay):
        self.animation_delay = animation_delay
        self.figure = plt.figure()
        self.subplot = self.figure.add_subplot(1, 1, 1)
        self.subplot.set_xlabel('x')
        self.subplot.set_ylabel('y')
        self.m1 = None
        self.m2 = None
        self.st = None
        self.mmid = None

    def plot_function(self, x1, x2, func, color='b'):
        x_range = np.linspace(x1, x2, 500)
        self.subplot.plot(x_range, func(x_range), color)

    def add_zero_line(self, x1, x2):
        self.subplot.plot([x1, x2], [0, 0], 'k.--')
        plt.draw()

    def update_edge_points(self, x1, y1, x2, y2):
        if self.m1:
            self.m1.remove()
        if self.m2:
            self.m2.remove()
        if self.st:
            self.st.remove()

        self.m1, = self.subplot.plot([x1, x1], [0, y1], 'mo--')
        plt.draw()

        self.m2, = self.subplot.plot([x2, x2], [0, y2], 'mo--')
        plt.draw()

        self.st, = self.subplot.plot([x1, x2], [y1, y2], 'k.-.')
        plt.draw()
        plt.pause(self.animation_delay)

    def clear_connecting_line(self):
        if self.st:
            self.st.remove()
            self.st = None
            plt.draw()

    def update_middle_point(self, x, y):
        if self.mmid:
            self.mmid.remove()

        self.mmid, = self.subplot.plot([x, x], [0, y], 'ks--')
        plt.draw()
        plt.pause(self.animation_delay)

    def clear_middle_point(self):
        if self.mmid:
            self.mmid.remove()
            self.mmid = None
            plt.draw()

class StandardMathLibrary:
    sin = math.sin
    cos = math.cos
    log = math.log

class NumpyMathLibrary:
    sin = np.sin
    cos = np.cos
    log = np.log

class SympyMathLibrary:
```

```
    sin = sympy.sin
    cos = sympy.cos
    log = sympy.log

def create_scroll_textbox(width: int, height: int):
    root = tk.Tk()
    root.title("Scroll Text Box")

    frame1 = tk.Frame(root)
    frame1.pack()

    # T = ScrolledText(root, height=h, width=w,wrap=WORD) # jeigu reikia scrolinti tik pagal h
    scrollbar_y = tk.Scrollbar(frame1, orient='vertical')
    scrollbar_y.pack(side=tk.RIGHT, fill=tk.Y)

    scrollbar_x = tk.Scrollbar(frame1, orient='horizontal')
    scrollbar_x.pack(side=tk.BOTTOM, fill=tk.X)

    textbox = tk.Text(frame1,
                      width=width, height=height,
                      yscrollcommand=scrollbar_y.set, xscrollcommand=scrollbar_x.set,
                      wrap=tk.NONE)
    textbox.pack()

    scrollbar_x.config(command=textbox.xview)
    scrollbar_y.config(command=textbox.yview)

    return textbox

def textbox_append_line(textbox: tk.Text, line: str):
    textbox.insert(tk.END, line + "\n")
    textbox.yview(tk.END)

def get_guess_accuracy(x1, x2, f1, f2, eps):
    x_abs_diff = np.abs(x1-x2)
    x_abs_sum = x1+x2
    f1_abs = np.abs(f1)
    f2_abs = np.abs(f2)
    if not np.isscalar(x1):
        x_abs_diff = sum(x_abs_diff)
        x_abs_sum = sum(x_abs_sum)
        f1_abs = sum(f1_abs)
        f2_abs = sum(f2_abs)

    if x_abs_sum > eps:
        return x_abs_diff/(x_abs_sum + f1_abs + f2_abs)
    else:
        return x_abs_diff + f1_abs + f2_abs

def scan_root_ranges(LF, x1, x2, step=0.1):
    x = x1
    while x < x2:
        if np.sign(LF(x)) != np.sign(LF(x+step)):
            yield (x, x+step)
        x += step

def grubus_R(LF_terms):
    return 1 + max(abs(ai) for ai in LF_terms[:-1]) / LF_terms[-1]

def mul_every_by(numbers: list[float], scalar: float):
    return list(map(lambda a: scalar*a, numbers))

def tikslesnis_R_range(LF_terms: list[float]):
    LF_terms = LF_terms.copy()
    if LF_terms[-1] < 0:
        # f(x) => -f(x)
        LF_terms = mul_every_by(LF_terms, -1)


    k = len(LF_terms) - max(i for i, a in enumerate(LF_terms[:-1]) if a < 0) - 1
    B = max(abs(a) for a in LF_terms[:-1] if a < 0)
    R_teig = 1 + (B / LF_terms[-1]) ** (1/k)

    # f(x) => f(-x)
    for i in range(1, len(LF_terms), 2):
        LF_terms[i] *= -1
```

```
    if len(LF_terms) % 2 == 0:
        # f(x) => -f(x)
        LF_terms = mul_every_by(LF_terms, -1)

    if any(True for a in LF_terms[:-1] if a < 0) > 0:
        B = max(abs(a) for a in LF_terms[:-1] if a < 0)
        k = len(LF_terms) - max(i for i, a in enumerate(LF_terms[:-1]) if a < 0) - 1
        R_neig = 1 + (B / LF_terms[-1]) ** (1/k)
    else:
        R_neig = 0

    return (R_neig, R_teig)

def roots_range(LF_terms: list[float]):
    R = grubus_R(LF_terms)
    R_neig, R_teig = tikslesnis_R_range(LF_terms)
    return (-min(R, R_neig), min(R, R_teig))

def get_coeffs(func):
    x_symbol = sympy.symbols("x")
    a: sympy.Poly = sympy.Poly(func(x_symbol), x_symbol)
    coeffs = list(map(float, a.coeffs()))
    coeffs.reverse()
    return coeffs

def solve_stygos(F, min_x, max_x, view_x, max_iterations=30, epsilon=1e-6, animation_delay=0.05):
    numpy_F = lambda x: F(NumpyMathLibrary, x)

    textbox = create_scroll_textbox(140, 20)
    textbox_append_line(textbox, "-------- Stygu metodas ------------")

    my_plot = MyPlot(animation_delay)
    my_plot.plot_function(view_x[0], view_x[1], numpy_F)
    my_plot.add_zero_line(view_x[0], view_x[1])

    textbox_append_line(textbox, "Saknies reiksmes tikslinimas Stygu metodu:")

    results = []

    for x1, x2 in scan_root_ranges(numpy_F, min_x, max_x):
        start_x1 = x1
        start_x2 = x2
        my_plot.subplot.plot([x1, x2], [0, 0], 'mo--')
        plt.draw()

        f1 = numpy_F(x1)
        f2 = numpy_F(x2)
        my_plot.update_edge_points(x1, f1, x2, f2)

        for i in range(1, max_iterations + 1):
            if f2 == 0:
                f2 += epsilon/2
            k = np.abs(f1 / f2)
            x_middle = (x1 + k * x2) / (1 + k)  # stygos nulio taskas
            middle_value = numpy_F(x_middle)

            my_plot.clear_connecting_line()
            my_plot.update_middle_point(x_middle, middle_value)

            if np.sign(f1) != np.sign(middle_value):
                x2 = x_middle
                f2 = middle_value
            else:
                x1 = x_middle
                f1 = middle_value

            my_plot.clear_middle_point()
            my_plot.update_edge_points(x1, f1, x2, f2)

            tksl = get_guess_accuracy(x1, x2, f1, f2, epsilon)
            textbox_append_line(textbox, f"{i:2d}. x1={x1:-20g}, x2={x2:-20g}, f1={f1:-20g}, f2={f2:-20g}, tikslumas={tksl:-
20g}")

            if tksl < epsilon:
                break
```

```
        tksl = get_guess_accuracy(x1, x2, f1, f2, epsilon)
        if tksl < epsilon:
            print(f"Isspresta: x={x1:g}, f={f1:g}, tikslumas={tksl:g}")
        else:
            print("Tikslumas nepasiektas")

        results.append([start_x1, start_x2, x1, tksl, i])

    return results

def solve_liestiniu(F, min_x, max_x, view_x, epsilon=1e-8, animation_delay=0.5):
    sympy_F = lambda x: F(SympyMathLibrary, x)
    numpy_F = lambda x: F(NumpyMathLibrary, x)

    x_symbol = sympy.symbols("x")
    DF_symbolical = sympy_F(x_symbol).diff(x_symbol)
    DF = sympy.lambdify((x_symbol), DF_symbolical)

    x=np.arange(view_x[0], view_x[1], 0.05); y=numpy_F(x)

    plt.xlabel("x"); plt.ylabel("y"); plt.plot(x, y, 'b'); plt.grid()

    results = []

    for x1, x2 in scan_root_ranges(numpy_F, min_x, max_x):
        start_x = (x1+x2)/2

        xi = start_x
        iterations = 0
        while np.abs(numpy_F(xi)) > epsilon:
            xi_bef = xi
            xi = xi - (1 / DF(xi)) * numpy_F(xi)
            iterations += 1

            plt.plot([xi_bef], numpy_F(xi_bef), 'ob')
            plt.plot([xi], [0], 'or')
            plt.plot([xi_bef, xi], [numpy_F(xi_bef), 0], 'r-')
            plt.plot([xi, xi], [0, numpy_F(xi)], 'g--')
            plt.draw()
            plt.pause(animation_delay)

        results.append((x1, x2, xi, np.abs(numpy_F(xi)), iterations))

    return results

def prefix_results(prefix, results):
    return map(lambda line: [*prefix, *line], results)

def main_1(F, G, G_range, F_view_x):
    results = []

    F_min_x, F_max_x = roots_range(get_coeffs(lambda x: F(StandardMathLibrary, x)))
    results.extend(prefix_results(["f(x)", "Stygų"    ], solve_stygos(F, F_min_x, F_max_x, F_view_x)))
    results.extend(prefix_results(["f(x)", "Liestinių"], solve_liestiniu(F, F_min_x, F_max_x, F_view_x)))

    results.extend(prefix_results(["g(x)", "Stygų"    ], solve_stygos(G, G_range[0], G_range[1], G_range)))
    results.extend(prefix_results(["g(x)", "Liestinių"], solve_liestiniu(G, G_range[0], G_range[1], G_range)))

    headers = ["Funkcija", "Metodas", "Skenavimo min", "Skenavimo max", "Kirtimo taškas","Tikslumas", "Iteracijos"]
    print(tabulate(results, headers, tablefmt="grid"))

def print_coeffs_desmos(coeffs):
    coeffs_strs = []
    for i, coeff in enumerate(coeffs):
        x_power = len(coeffs)-i-1
        x_power_str = "^".join(c for c in str(x_power))
        coeffs_strs.append(f"({coeff} * x^{x_power_str})")
    print(" + ".join(coeffs_strs))

def main_2(H, H_range, epsilon=1e-4, animation_delay=0.05):
    standard_H = lambda x: H(StandardMathLibrary, x)
    numpy_H = lambda x: H(NumpyMathLibrary, x)
    sympy_H = lambda x: H(SympyMathLibrary, x)

    my_plot = MyPlot(animation_delay)
```

```
    my_plot.plot_function(H_range[0], H_range[1], numpy_H)
    plt.pause(animation_delay)

    x,f,fp,df=sympy.symbols(('x','f','fp','df'))

    f=sympy_H(x)
    x0=(H_range[0]+H_range[1])/2

    fp=f.subs(x,x0)
    N = 1

    while True:
        f=f.diff(x)
        fp=fp+f.subs(x,x0)/math.factorial(N)*(x-x0)**N;
        N += 1

        a=sympy.Poly(fp,x)
        if abs(a(H_range[0]) - standard_H(H_range[0])) < epsilon and abs(a(H_range[1]) - standard_H(H_range[1])) < epsilon:
            break

        if N % 5 == 0:
            my_plot.plot_function(H_range[0], H_range[1], sympy.lambdify(x, fp, 'numpy'), 'r--')
            plt.pause(animation_delay)

    a=sympy.Poly(fp,x)

    print(N)
    print_coeffs_desmos(a.all_coeffs())
    my_plot.plot_function(H_range[0], H_range[1], sympy.lambdify(x, fp, 'numpy'), 'r--')
    plt.pause(animation_delay)

# Variantas: 20
main_1(
    F=(lambda m, x: 1.34 * (x**4) - 16.35 * (x**3) + 65.13 * (x**2) - 83.45 * (x**1) - 3.27),
    G=(lambda m, x: m.sin(x) - m.log(x)/2 + 0.1),
    G_range=(0.1, 10),
    F_view_x=(-0.5, 5)
)

main_2(
    H=(lambda m, x: -8*m.sin(x) + 32*m.sin(10*x) - 10),
    H_range=(0, 1)
)

input("Baigti darba! Wooo! Press 'ENTER'")
```